Fixed Power Allocation with Nulling for TDD-Based Cellular Uplink

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***Abstract*— We propose two fixed power allocation schemes with nulling (FPA-N1 and FPA-N2) for time division duplex (TDD) based cellular uplink according to the location of mobile stations (MSs). In the FPA-N1 scheme, MSs located near a base station (BS) do not transmit data when the wireless channels between the MSs and their home cell BS experience deep fading. In the FPA-N2 scheme, MSs located near cell boundaries do not transmit data when the wireless channels between the MSs and neighboring cell BSs cause high interference channel gain because, in this case, their data transmission may induce large interference to neighboring cells. Numerical results show that the proposed power allocation scheme improves the uplink capacity in cellular networks.**

***Index Terms*— Power allocation, fading channels, inter-cell interference.**

I. INTRODUCTION

CHANNEL capacity is an important performance metric for digital communication systems. Studies on Shannon capacity in fading channels have been extensively done [1]–[3]. In particular, the channel capacity with channel state information (CSI) at both the transmitter and receiver was derived in [4] and it is achieved when the transmitter adapts its power and data rates to the channel variation. The optimal power allocation is known as a *water-filling (WF)* scheme in time, which is analogous to the water-filling used to achieve the optimal capacity on frequency-selective fading channels [5]. However, the WF requires the information about instantaneous channel gain from the receiver and the power allocation scheme based on the WF is generally more complex than the fixed power allocation scheme.

A wireless link generally suffers from fading as well as interference. When we consider an uplink in an interference-limited cellular system with multiple base stations (BSs) and mobile stations (MSs), transmitting MSs in cell boundaries induce the inter-cell interference to neighboring BSs. Information-theoretic analysis of cellular uplink with various multiple access schemes was given in [6] and the authors revealed the inter-cell interference is one of the major factors which degrade the system performance. Interference management schemes using radio resource allocation was proposed in [7] and coordinating multiple BSs was proposed to improve the performance of MSs in cell-boundary [8]. However, it is difficult to implement these schemes for mitigating inter-cell interference. Uplink scheduling or coordination is even more difficult than the case of downlink because the interference sources are MSs rather than fixed BSs. In this letter, we propose two fixed power allocation schemes with nulling (FPA-N1 and FPA-N2) for improving the system capacity of the uplink in TDD-based cellular networks.

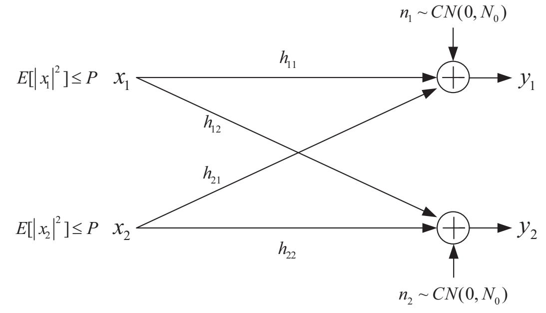

Fig. 1. System model of uplink cellular networks with two BSs.

II. SYSTEM MODEL

We consider a cellular network with two BSs each of which serves non-overlapping MSs. We assume that every node has one antenna and the channel is in frequency-flat fading. Fig. 1 shows the system model of an uplink cellular network with two BSs. x_i indicates the transmitted signal of an MS scheduled in cell i and y_j denotes the received signal at the BS of cell j $(1 \le i, j \le 2)$. The received signals during a time slot are expressed as:

$$y_1 = h_{11}x_1 + h_{21}x_2 + n_1 \tag{1}$$

$$y_2 = h_{12}x_1 + h_{22}x_2 + n_2, \tag{2}$$

where h_{ij} and n_i represent the channel coefficient from an MS scheduled in cell i to the BS of cell j and the thermal noise at the BS i, respectively. The wireless channel is assumed to be Rayleigh distributed, i.e., $h_{ij} \sim \mathcal{CN}(0, \sigma_{ij}^2)$. The variance of the complex gaussian random variable, σ_{ij}^2, implies the mean of the channel gain between the i-th MS and the j-the BS.

We divide MSs into two groups depending on their locations, i.e., MSs in cell boundaries and MSs near their serving BSs. Time slots are also divided into two MS groups. Hence, for a given time fraction, power allocation for the two MS groups can be decoupled. We propose two fixed power allocation schemes, which are simple and easily implementable.

III. CAPACITY ANALYSIS OF THE PROPOSED SCHEME

A. When MSs are located near a BS (Scenario I)

In this scenario, the interference channel gains are nearly zero, i.e., σ_{12}^2 and σ_{21}^2 are nearly zero. The received signal

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from each MS only suffers from fading. For a fixed power allocation scheme, the average capacity of the MS in the i-th cell is given as [1]:

$$\begin{aligned} C_{FPA} &= \int_0^\infty \log_2(1+\gamma_i)\cdot\frac{1}{\overline{\gamma_i}}\exp\left(-\frac{\gamma_i}{\overline{\gamma_i}}\right)d\gamma_i \\ &= \log_2(e)\cdot\exp\left(1/\overline{\gamma_i}\right)E_1\left(1/\overline{\gamma_i}\right), \end{aligned} \tag{3}$$

where $\gamma_i = P|h_{ii}|^2/N_0$ and $E[\gamma_i] = \overline{\gamma_i}$. The term $\overline{\gamma_i}$ indicates the average received signal-to-noise ratio (SNR) and the exponential integral function $E_1(x)$ is defined as $E_1(x) = \int_x^\infty e^{-t}t^{-1}dt$ [9].

In this scenario, we propose that MSs do not transmit data when the received SNR is less than a given threshold. If the received SNR is larger than the threshold, then MSs transmit data with a fixed power level. The proposed fixed power allocation rule with nulling (FPA-N1) is expressed as:

$$P(\gamma_i) = \begin{cases} \frac{1}{\alpha}\cdot P, & \gamma_i \ge \gamma_0 \\ 0, & \gamma_i < \gamma_0 \end{cases} \tag{4}$$

where $\alpha = \int_{\gamma_0}^\infty 1/\overline{\gamma_i}\exp\left(-\gamma_i/\overline{\gamma_i}\right)d\gamma_i = e^{-\frac{\gamma_0}{\overline{\gamma_i}}}$ and γ_0 is a given threshold. When the proposed FPA-N1 is used, for given γ_0 and $\overline{\gamma_i}$, the average capacity of the MS in the i-th cell is given by

$$\begin{aligned} C_{FPA-N}(\gamma_0,\overline{\gamma_i}) &= \int_{\gamma_0}^\infty \log_2\left(1+\frac{\gamma_i}{\alpha}\right)\cdot\frac{1}{\overline{\gamma_i}}\exp\left(-\frac{\gamma_i}{\overline{\gamma_i}}\right)d\gamma_i \\ &= \frac{1}{\ln 2}\cdot\int_{\gamma_0}^\infty f(\gamma_i)g^{'}(\gamma_i)d\gamma_i, \end{aligned} \tag{5}$$

where $f(\gamma_i) = \ln(1+\gamma_i/\alpha)$ and $g^{'}(\gamma_i) = 1/\overline{\gamma_i}\exp(-\gamma_i/\overline{\gamma_i})$. If we use the method of integration by part and Eq. (2.325.1) in [9], Eq. (5) is expressed as

$$\begin{aligned} & C_{FPA-N}(\gamma_0,\overline{\gamma_i}) = \\ & \frac{1}{\ln 2}\cdot\left[\alpha\ln\left(1+\frac{\gamma_0}{\alpha}\right)+e^{\alpha/\overline{\gamma_i}}E_1\left(\frac{1}{\overline{\gamma_i}}(\alpha+\gamma_0)\right)\right]. \end{aligned} \tag{6}$$

The optimal threshold can be chosen by maximizing Eq. (6) for a given $\overline{\gamma_i}$ through the numerical differentiation and the transmit power is determined by the optimal threshold. In the proposed FPA-N1, MSs only need to know whether the received SNR is larger than the threshold or not.

B. When MSs are located in cell boundaries (Scenario II)

In this scenario, we assume that all σ_{ij}^2 have an identical value of 1. When the received signal suffers from inter-cell interference as well as fading, the received signal-to-interference-plus-noise ratio (SINR), γ_i, is given by $|h_{ii}|^2P/\left(|h_{ij}|^2P+N_0\right) = |h_{ii}|^2/\left(|h_{ij}|^2+1/\rho\right)$, where $\rho = P/N_0$. Then, the probability density function (PDF) of the received SINR is given as [10]:

$$f_{\gamma_i}(\gamma_i) = \frac{e^{-\gamma_i/\rho}}{(1+\gamma_i)^2}\cdot\left(\frac{1}{\rho}(1+\gamma_i)+1\right). \tag{7}$$

For the conventional fixed power allocation scheme, the average capacity of the MS in the i-th cell is given as:

$$\begin{aligned} \widetilde{C}_{FPA} &= \int_0^\infty \log_2(1+\gamma_i)\cdot f_{\gamma_i}(\gamma_i)d\gamma_i \\ &= \frac{e^{1/\rho}}{\ln 2}\left[\int_1^\infty\frac{\ln x}{x}\cdot\frac{1}{\rho}e^{-\frac{x}{\rho}}dx+\int_1^\infty\frac{\ln x}{x^2}\cdot e^{-\frac{x}{\rho}}dx\right], \end{aligned} \tag{8}$$

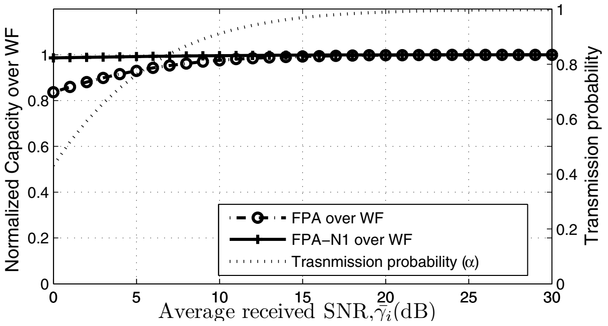


Fig. 2. Normalized capacities (left vertical axis) of the FPA scheme and the FPA-N1 scheme by the capacity of the WF scheme in Scenario I and the transmission probability (right vertical axis) of the proposed FPA-N1 when the optimum threshold is used.

where $x = 1+\gamma_i$. We define that $f(x) = \ln x/x$ and $g^{'}(x) = 1/\rho\exp(-x/\rho)$ and use the method of integration by part and Eq. (2.325.2) in [9], Eq. (8) is expressed as:

$$\widetilde{C}_{FPA} = \log_2(e)\cdot\left[1-\frac{1}{\rho}e^{1/\rho}E_1(1/\rho)\right]. \tag{9}$$

Note that the average capacity approaches a value of $\log_2(e)$ as the ρ value increases. We now propose that MSs do not transmit data when the interference channel gain between MSs and their interfering BS is larger than some threshold. If the interference channel gain is less than the threshold, then MSs transmit data with the fixed power. We assume that each MS knows the interference channel gain between the MS and its neighboring cell BSs. The interference channel gains can be obtained by the pilot signals from the neighboring cells in time division duplex (TDD) systems. The proposed fixed power allocation rule with nulling (FPA-N2) is expressed as:

$$P(h_{ij}) = \begin{cases} P, & |h_{ij,i\neq j}|^2 \le g_0 \\ 0, & |h_{ij,i\neq j}|^2 > g_0, \end{cases} \tag{10}$$

where g_0 indicates a threshold of the interference channel gain. The average capacity of the MS in the i-th cell is derived in the Appendix.

IV. NUMERICAL EXAMPLES

Fig. 2 compares the normalized capacities of the FPA scheme and the FPA-N1 scheme over the capacity of the WF scheme in Scenario I. The FPA-N1 scheme yields quite a similar capacity to that of the WF scheme from low SNR values to high SNR values, while the capacity of the conventional FPA scheme approaches the capacity of the WF scheme for high SNR values. In addition, the transmission probability of the proposed FPA-N1 scheme increases as the SNR values increase when the optimum threshold is applied. The value of optimal threshold also increases as the average SNR increases. The optimum threshold is computed according to varying SNR values and saved for a given fading statistics, and it is applied to update the power allocation rule in Eq. (4).

Fig. 3(a) compares the capacity of the FPA-N2 scheme with that of the FPA scheme in Scenario II. The capacity of the conventional FPA scheme is saturated to $\log_2(e) \simeq 1.4427$ as the average transmit SNR value, ρ, increases, while the proposed FPA-N2 keeps increasing as ρ increases. Hence, the capacity difference between the FPA and the FPA-N2 increases as ρ increases. Note that the FPA-N2 scheme operates in a distributed manner since the transmission is determined by each MS according to the interfering channel gains which can be measured by the pilot signals from other cell BS. Fig. 3(b) shows the optimum threshold of the interfering channel gains for achieving the optimum throughput and the transmission probability of the proposed FPA-N2 scheme in Scenario II.

V. CONCLUSIONS

We proposed two fixed power allocation schemes with nulling (FPA-N1 and FPA-N2) for TDD-based cellular uplink and analyzed its performance in terms of Shannon capacity. The proposed FPA-N1 and FPA-N2 schemes are simple and operated in a distributed manner. The proposed FPA-N1 scheme yields similar performance to the optimum WF schemes where MSs are located near the BSs. When the MSs are located near cell boundaries, the proposed FPA-N2 scheme also yields much better performance than that of the conventional FPA scheme. In the scenario for cell boundary MSs, we proposed that the MSs determine their data transmissions according to varying interference channel gains. However, in this scenario, the better scheme is to determine data transmissions considering their own channel gains as well as the interfering channel gains. We will leave this scheme for further work.

APPENDIX

Based on this power allocation strategy, the received SINR is expressed as $\gamma_i = z/\left(1/\rho + y\right)$, where the PDF of z is equal to e^{-z} and the PDF of y is given as:

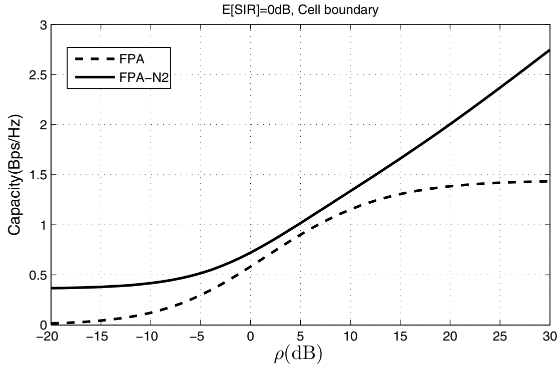
$$f_Y(y) = \begin{cases} (1-\beta)\delta(y), & y = 0 \\ e^{-y}, & 0 < y < g_0 \\ 0, & y \geq g_0, \end{cases} \tag{11}$$

where $\beta = \int_0^{g_0} e^{-\gamma_i} d\gamma_i$ for a given threshold and $\delta(\cdot)$ indicates the delta function. y is a mixed random variable because it has a probability mass when $y = 0$. Conditioning on the interference channel gain, the PDF of the received SINR in the proposed scheme can be expressed as:

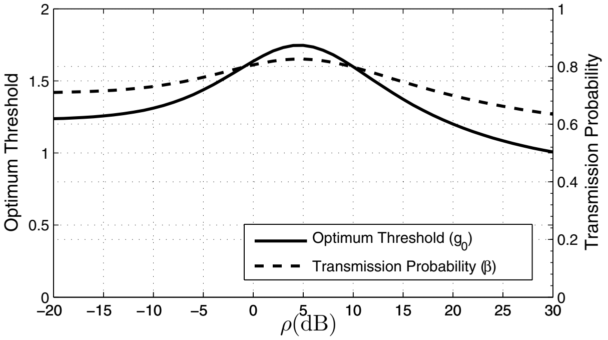
$$\begin{aligned} \widetilde{f}(\gamma_i) &= \int_0^\infty f_{\gamma_i|y}\left(\gamma_i|y\right) f_Y(y)dy \\ &= \int_0^{g_0} f_{\gamma_i|y}\left(\gamma_i|y\right) f_Y(y)dy + f_{\gamma_i|y}\left(\gamma_i|y=0\right)\Pr(y=0) \\ &= \int_0^{g_0} \left(\frac{1}{\rho} + y\right) e^{-(1/\rho + y)\gamma_i} \cdot e^{-y}dy + \frac{e^{-\frac{\gamma_i}{\rho}}}{\rho} \cdot (1-\beta) \\ &= \frac{e^{-\frac{\gamma_i}{\rho}}}{\rho(\gamma_i+1)} \cdot \left[1 - e^{-(\gamma_i+1)g_0} - g_0\rho e^{-(\gamma_i+1)g_0}\right] \\ &\quad + \quad \frac{e^{-\frac{\gamma_i}{\rho}}}{(\gamma_i+1)^2}\left[1 - e^{-(\gamma_i+1)g_0}\right] + \frac{e^{-g_0}}{\rho} e^{-\frac{\gamma_i}{\rho}}. \end{aligned} \tag{12}$$

The average capacity of the MS in the i-th cell is given by [9]

$$\begin{aligned} \widetilde{C}_{FPA-N}(g_0, \overline{\gamma_i}) &= \beta \int_0^\infty \log_2\left(1+\gamma_i\right) \cdot \widetilde{f}(\gamma_i)d\gamma_i \\ = \beta \log_2(e) \cdot \Big[1 - 1/\rho e^{1/\rho} E_1(1/\rho) - 1/e^{g_0} \\ &+ \tau e^{1/\rho} E_1(\tau) + e^{1/\rho}/e^{g_0} E_1(1/\rho)\Big], \end{aligned} \tag{13}$$



(a) Capacities of the proposed FPA-N2 and the conventional FPA schemes



(b) Optimum threshold (left vertical axis) and the transmission probability (right vertical axis) of the proposed FPA-N2 scheme

Fig. 3. Capacity, optimum threshold, and transmission probability of the FPA-N2 scheme in Scenario II.

where $\tau = \frac{1+g_0\rho}{\rho}$. Since β indicates the transmission probability in the proposed scheme, the average power consumption is equal to βP and, thus, the average transmit SNR, ρ, is equal to $\beta P/N_0$. The optimal threshold is also chosen by maximizing Eq. (13) for a given ρ through the numerical differentiation.

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